ME140A - Homework 2

October 8, 2022

Due by 11:59PM, Oct 14th, by email to ameiburg@ucsb.edu. Collaboration is encouraged!

1 Problem 1 - Double Integral

Consider the double integral,

$$\int_{x=3}^{4}\int_{y=2}^{3} xy - y^3 + \frac{x}{y}\,dy\,dx$$

1.1 (a)

Compute the exact integral. (Wolfram Alpha or similar is fine.)

1.2 (b)

Compute a numerical integral with (the simple, $n = 1$) Simpson's 1/3 rule in each direction. Compute the relative accuracy.

2 Problem 2 - Non-rectangular integral

In lecture, we discussed two ways of computing a double integral on a rectangular region. For an integral like,

$$\int_{x=a}^{b}\int_{y=c}^{d} f(x,y)\,dy\,dx$$

we can either,

1. Divide into a 2D grid and use a simple rule (like 2D trapezoidal) on each square
2. Define $F(y) = \int_{x=a}^{b} f(x,y)\,d$, compute F at a given point using a 1D integration rule, and then integrate F using an integration 1D integration rule.

However, we didn't explicitly talk about non-rectangular integrals.

2.1 (a)

How would you integrate an expression like

$$\int_{x=1}^{2} \int_{y=sin(x)}^{x^2} \frac{y^2}{1+e^x}\, dy\, dx$$

You're not expected to write code or evaluate this, just explain the integration approach.

2.2 (b)

How would you integrate $f(x, y)$ over the unit circle, that is, the set of points (x, y) where $x^2 + y^2 \leq 1$? Again, just explain in words.

3 Problem 3 - Numerical derivative

Define

$$f(x) = \frac{sin(x)}{x^2 + 1}$$

3.1 (a)

Compute the exact derivative of at $x = 0.2$.

3.2 (b)

Numerically compute the derivative at $x = 0.2$, using:

1. Forward difference: $f'(x) \approx \frac{f(x+h)-f(x)}{h}$
2. Central difference: $f'(x) \approx \frac{f(x+h)-f(x-h)}{2h}$
3. Five-point stencil: $f'(x) \approx \frac{-f(x+2h)+8f(x+h)-8f(x-h)+f(x-2h)}{12h}$

With $h = 0.1$, $h = 0.01$, and $h = 0.001$. How did the errors compare?